The Nature of Models

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Summary

Theories stand on their own feet, and rely on no analogies.

Models stand on someone else's feet. They are metaphors that explain the world we don't understand in terms of worlds we do.

Models tell you what something is more or less like; theories try to tell you what something is.

The equations of physics and finance resemble each other syntactically, but their semantics is very different.

Financial models are **not** theories; they are analogies, idealizations that always sweep dirt under the rug, and good models and good modelers have an obligation to make the the dirt explicit.

Ways of Knowing

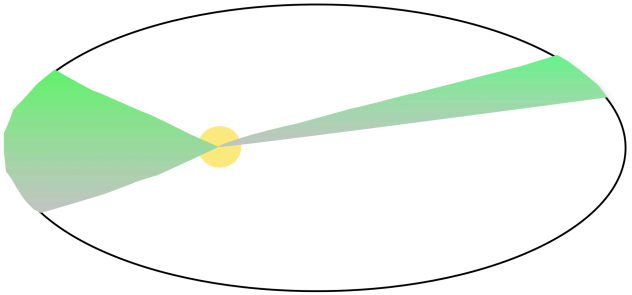
- How do we approach understanding the world?
- The great triumph at the dawn of modern science: the understanding of gravitation and motion
- For millennia after the Greeks, scientists' prejudices led them describe all planetary movements in terms of circles about a stationary earth.
- But the motion of a planet, as seen from the orbiting earth itself, is too complicated for a single circle and so it needs circles moving on circles moving on
- Eventually, Galileo pointed out that the earth wasn't stationary, that the earth and planets orbited the sun, and that the planets' weird apparently retrograde motions were not intrinsically theirs but rather a consequence of their being observed from the moving earth.

Kepler's Laws

- In the early 1600s Kepler examined the data on planetary positions and formulated his three astonishing laws of planetary motion:
  - planets move in ellipses about the sun;
  - the line between the Sun and a planet sweeps out equal areas in equal times;
  - the square of the orbital period is proportional to the cube of the distance from the sun.

Kepler's 2nd Law

- For insight into the miracle of discovery, think about Kepler's second law.
- The line between the Sun and a planet sweeps out equal areas in equal times;



- This deep symmetry of planetary motion implies that the closer the planet to the sun, the more rapidly it moves, as shown.
- *There was no line* between a planet and the sun for Kepler to observe. His data consisted of planetary positions in the night sky.
- How then did he decide to describe the motion of the planets in terms of an invisible imaginary line?
- No one knows exactly, but it involved long immersion, struggle, and associative, and then - Aha! - intuition, followed by checking the data.

Newton

- Kepler's laws described the patterns of the planets, but not their causes.
- Newton found a cause; he showed that Kepler's laws were a mathematical consequence of Newton's own theories:
  - *Theory* of gravitation (the inverse square law of attraction) and
  - Laws of motion (Force = mass times acceleration).
- How did Newton discover his theories? For sure, the orbiting planets and falling apples didn't announce the laws that drove them.

Modes of Understanding 1: Intuition

- It takes intuition to discover the nature of the world.
  - Kepler, Newton, Ampere, Maxwell, Einstein, Dirac …
- Intuition may sound casual but it takes intimate knowledge of the world acquired by careful observation and painstaking effort.
- Keynes on Newton:
  I fancy his pre-eminence is due to his muscles of intuition being the strongest and most enduring with which a man has ever been gifted …I believe that the clue to his mind is to be found in his unusual powers of continuous concentrated introspection … His peculiar gift was the power of holding continuously in his mind a purely mental problem until he had seen straight through it.
- Maxwell on Ampére
  "We can scarcely believe that Ampere really discovered the law of action by means of the experiments which he describes. We are led to suspect, what, indeed, he tells us himself, that he discovered the law by some process which he has not shown us, and that when he had afterwards built up a perfect demonstration, he removed all traces of the scaffolding by which he had built it."
- The observer becomes so close to the object (or person) observed that he begins to experience their existence from both outside and inside them. Intuition is a merging of the observer with the observed.

Modes of Understanding 2: Theories

- Theories are deep descriptions of the laws of the world.
- They can be right, partially right or totally wrong.
  - What all theories have in common is that, like God's voice to Moses in the desert, they proclaim:

I am what I am.

- Theories stand on their own feet. They are not analogies, but facts.
- Newton's laws have been supplanted by Einstein's, but that doesn't mean that Newton is an approximation to Einstein. Both Newton's and Einstein's are theories.
- Newton is to Einstein as cursive is to typing, or as navigation by the stars is to the Global Positioning System.
- Two different approaches reach the same end by different means, with different accuracies. One doesn't approximate the other. Both are theories that describe the facts.
- Maxwell's Theory for light

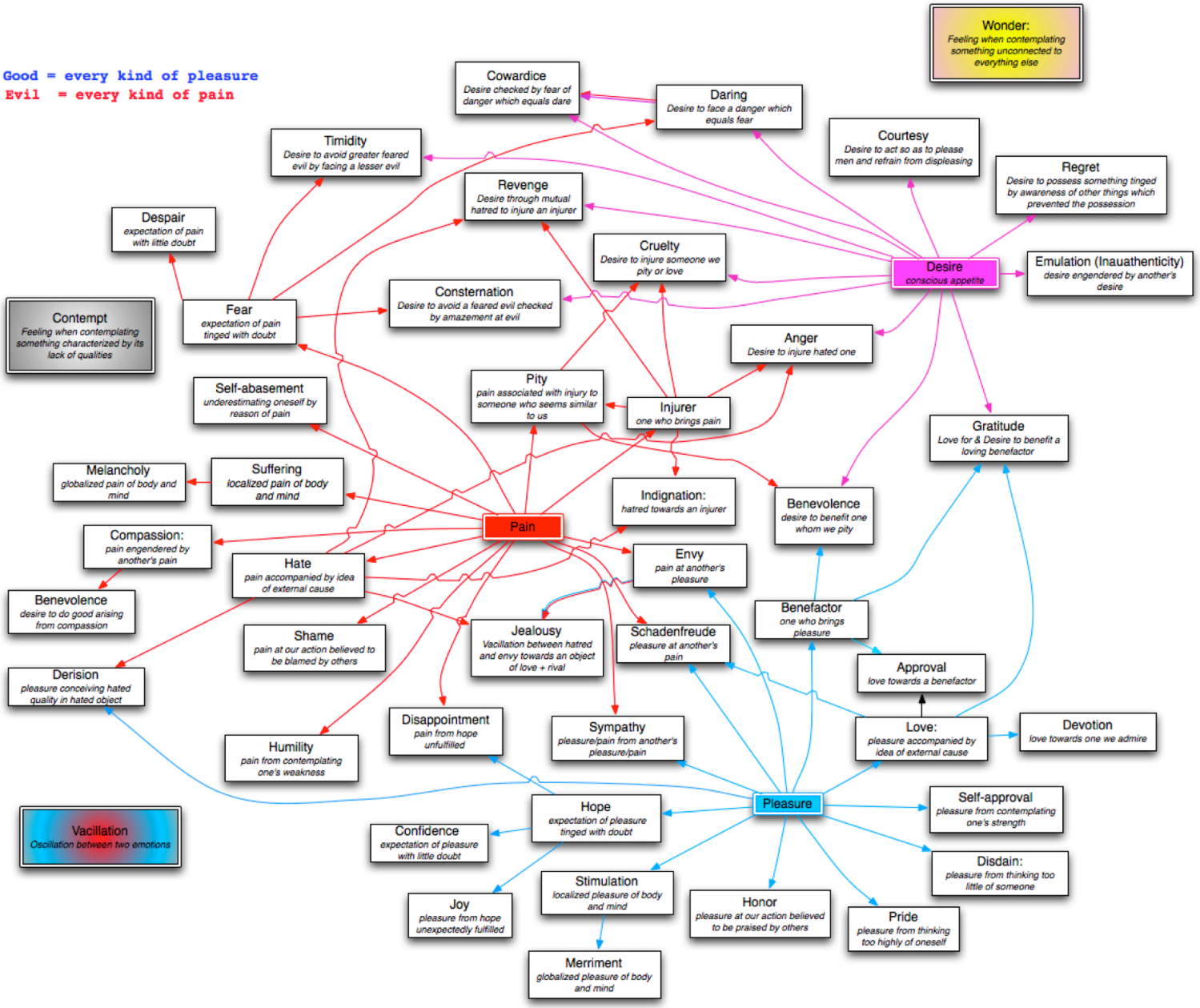
$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{D} = \rho$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- The Dirac Theory for the electron: $(-i\partial\!\!\!/ + m)\psi = 0$
- Goethe: One day we will realize that every fact is really a theory.

A Non-Physical Theory …Spinoza's Theory of Emotions as Derivatives

- Spinoza's treats emotions like Euclid treats geometry: emotions are derivatives.
  - Primitives are Desire, Pleasure, Pain.
- *Good* is everything that brings pleasure, and *Evil* is everything that brings pain.
- Love: Pleasure associated with an external object.
- Hate: Pain associated with an external object.
- Envy: Pain at another's Pleasure.
- Hope: Expectation of future Pleasure tinged with doubt.
- Fear: Expectation of future Pain.
- Cruelty: Desire to inflict Pain on a someone Loved.
- Three more primitives:
  - Vacillation, Wonder, Contempt.

Spinoza's Theory of Emotions as Derivatives



Modes of Understanding 3: Models

- *Sleep is the interest that we have to pay on the capital which is called in at death; and the higher the rate of interest and the more regularly it is paid, the further the date of redemption is postponed* – Arthur Schopenhauer
- Metaphors are an insight that something is like something else.
- A *model* is a metaphor: it compares something we don't understand to something we already do.
- The famous *liquid drop model* of the atomic nucleus pretends that the nucleus is a drop of water that can vibrate and rotate and even fission into two. Useful, picturesque, but not entirely true.
- Similarly, the *Black-Scholes financial option model* compares the uncertain movement of stock prices to the diffusion of smoke from a cigarette tip. Useful, up to a point -- but *not fact.*
- Theories tell you what something is. Models merely tell you what something is more or less like.
- Models are metaphors, graven images of reality but not reality itself.
- Models are analogies whose incautious use can unleash all the dangers of idolatry that God warned against in the second commandment.

Modes of Understanding 4: Data and Statistics

- There's one final mode of understanding: the statistical analysis that lies behind Big Data.
- Statistics seeks to find past tendencies and correlations in data.
- Often people assume they will persist.
- But, in a famous unattributed phrase, correlation does not imply causation.
- Big Data is useful, but is not a replacement for the classic ways of understanding the world.
- Data has no voice. There is no "raw" data. Choosing what data to collect takes insight; making good sense of it requires the classic method.
- You still need a model, a theory, or intuition to find a cause.
- Wittgenstein:
  "Philosophy is a battle against the bewitchment of our intelligence by means of language"
  I take that to mean that language can deceive our natural intuition, and we need philosophy to reclaim it.
- In a similar sense, science is a battle against the smothering of our intelligence by data.

Financial Models

Models in Finance

- Physics has theories and models. Finance has only models.
- The point of a model is not usually divination, which rarely works.
- A typical valuation model: apartment pricing & calibration.
- Models transform intuitive linear quantities into nonlinear dollar values.
  - Price per square foot to apartment price.
  - Future yield to bond price.
  - Future volatility to option price.
- Models interpolate from liquid prices to illiquid ones.
- Models in finance calibrate the future, predict the present.
- Models are used to ranks securities by value on a 1-D scale.

The One Law of Financial Modeling

- If you want to know the value of a (*target*) financial security, use the known price of another (*replicating)* portfolio of securities that's as similar to it as possible.
- Any two securities with identical future payoffs, *no matter how the future turns out*, should have identical current prices.
- To build a model:
  - Specify all future scenarios (science, reductive).
  - Prove similarity of payoffs under those scenarios (engineering, constructive).
- Newton to Mechanical Engineering
  Maxwell to Electrical Engineering
  ???? to Financial Engineering
- Brownian motion is a theory for dust particles, a model for stock prices

The Right Way to Use Valuation Models:

- Make use of replication, from best to worst:
  - Static, dynamic (Black-Scholes Merton), risk, more qualitative
- Avoid axiomatization and axioms. The world doesn't satisfy them.
- In physics it pays to drop down deep, use fundamental variables, formulate a principle, then come back up again.
- In finance, shallow is better. Use vulgar/market variables in a sophisticated way.
- Sweep dirt under the rug, but tell people about it.
- Think of models as *Gedanken* experiments.

The Financial Modelers' Manifesto

- I will remember that I didn't make the world, and it doesn't satisfy my equations.
- Though I will use the models I or others create to boldly estimate value, I will always look over my shoulder and never forget that the model is not the world.
- I will not be overly impressed by mathematics. I will never sacrifice reality for elegance without explaining to its end users why I have done so.
- I will not give the people who use my models false comfort about their accuracy. I will make the assumptions and oversights explicit to all who use them.
- I understand that my work may have enormous effects on society and the economy, many beyond my apprehension.
- MODELERS OF ALL MARKETS, UNITE!
  You have nothing to lose but your illusions.

Conclusion

- The solution to our financial crises will not lie in mathematics
- *If a fool would persist in his folly he would become wise.*
- A little hubris is good.
- Catastrophes strike when hubris evolves into idolatry. Somewhere between these two extremes, a little north of common sense but still south of idolatry, lies the wise use of models.